

1 Curriculum Vitæ

1.1 Personal information

Name: Aurelio SPADOTTO
Date and place of birth: 9 July 1997, Milano (Italy)
Nationality: Italian
Degree: PhD in Mathematics and Modelling

1.2 Current position

since 2025



Postdoctoral researcher

University of Montpellier
Institut Montpellierain Alexander Grothendieck (IMAG, UMR 5149)

Research topic: *Development of hybrid numerical schemes for Friedrichs systems*

Funding: ERC Synergy NEMESIS, project number 101115663

2024 – 2025



ATER (temporary teaching and research position)

University of Montpellier

1.3 Education

2021 – 2025



PhD in Mathematics and Modelling

University of Montpellier
Institut Montpellierain Alexander Grothendieck (IMAG, UMR 5149)

Thesis: *Numerical Simulation of Red Blood Cell Electrodeformation*

Advisors: Simon MENDEZ
Daniele DI PIETRO

Examiners: Lisl WEYNANS
Alexei LOZINSKI

Thesis committee president: Vincent MOUREAU

Other members: Jérôme DRONIOU

Funding: Region Occitanie, doctoral grant 2021

2019 – 2021



Master's degree in "Mathematical Engineering" - Scientific Computing track

Politecnico di Milano

2016 – 2019



Bachelor's degree in Mathematical Engineering

Politecnico di Milano

1.4 Computer skills

- **Languages:** Fortran 90 (scientific computing), C++ (scientific computing), Python (scientific computing and data visualization);
- **DevOps:** make, CMake, git;
- **3D Visualization:** paraview;
- **Document preparation:** \LaTeX .

2 Publications

2.1 Peer-reviewed journal articles

- DI PIETRO D., MENDEZ S. AND SPADOTTO A.
A discrete de Rham discretization of interface diffusion problems with application to the Leaky Dielectric Model.
Journal of Computational Physics, Vol. 530 (2025), pg. 113920
<https://doi.org/10.1016/j.jcp.2025.113920>
<https://arxiv.org/abs/2409.1/5042>

2.2 Submitted articles available as preprints

- DI PIETRO D. AND SPADOTTO A.
Hybrid Methods for Friedrichs Systems with Application to Scalar and Vector Diffusion-Advection Problems.
<https://arxiv.org/abs/2602.10890>(2026).

2.3 Articles in preparation

- DI PIETRO D., MENDEZ S. AND SPADOTTO A.
Numerical simulation of biological capsules based on polygonal meshes fitted along the membrane.
- DI PIETRO D. AND SPADOTTO A.
Hybrid Methods for 2-field Friedrichs Systems.

3 Conference and workshop participation

3.1 Invited presentations at conferences

- 2025 | Int. Conf. on Comput. Methods for Coupled Problems in Science and Eng., **Villasimius** ([slides](#))
- 2024 | POEMS24: Polytopal Element Methods in Mathematics and Engineering, **Paris** ([poster](#))
CJC-MA2024: Young Researchers Conference in Applied Mathematics, **Lyon** ([poster](#))
JMBS, Maths Bio Health Days, **Nantes** ([slides](#))
ECFD7: Extreme CFD Workshop, 7th edition, **Merville-Franceville** ([report](#), [sub-project N6](#))
- 2023 | Dynamics of Capsules, Vesicles and Cells in Flow **Compiègne** ([program](#))
ECFD6: Extreme CFD Workshop, 6th edition **Merville-Franceville** ([report](#), [sub-project 2](#))

3.2 Seminars and working groups

- 2026 | Applied mathematics seminar, **Nantes** ([slides](#))
- 2025 | ACSIOM team seminar (PDEs, numerical analysis) at IMAG, **Montpellier** ([slides](#))
NEMESIS working group (numerical analysis) at IMAG, **Montpellier** ([slides](#))

3.3 Event organisation and seminar management

- 2026 | Member of the organizing committee of the workshop "Towards Polytopal meshes in GMSH", **Montpellier** ([program](#))
- 2023-24 | Co-organization of the PhD students' seminar at IMAG, **Montpellier** ([program](#))

4 Teaching activity

The teaching activities listed below were carried out at the Faculty of Science of the University of Montpellier within the Department of Mathematics. They were performed in part as additional missions during the PhD and in part with an ATER position.

	U.E. (TD)	Level	Hours
2024/25	Scientific Reasoning	L1 Biology	75h
	Mathematical Tools 1	L1 PCSI	90h
	Mathematical Tools 2	L1 PCSI	27h
			***** TOT *****
			192h
2023/24	Computational Methods	L1 Biology	16h
	Mathematical Tools 1	L1 PCSI	16h
			***** TOT *****
			32h
2022/23	Computational Methods	L1 Biology	32h
			***** TOT *****
			32h

4.1 Course contents

- **Mathematical Tools 1 (L1 Physics/Chemistry/Engineering Sciences (PCSI), practicals, 106 total hours)** : elements of set theory and logic (statements, logical connectors, quantifiers, proof by contradiction); topology of sets in \mathbb{R} ; basic vocabulary on the concept of a function; analysis of functions of one real variable: limits, continuity, differentiation; Taylor expansions and application to limit computations; optimization of functions of one real variable; Riemann integration, fundamental theorem of calculus and standard techniques for computing integrals; introduction to first-order ordinary differential equations: separation of variables, linear ODEs with constant coefficients, Cauchy problems, modelling examples from chemistry and ecology.
- **Scientific Reasoning (L1 Biology, practicals, 90 total hours)**: elements of set theory and modelling of a random experiment; probability calculations for simple experiments, basic combinatorics; independent events, conditional probability, Bayes theorem; random variables: distribution, expectation, variance, median; probability laws on finite sets: uniform, Bernoulli, binomial; laws on countable sets: geometric, Poisson;
- **Computational Methods (L1 Biology, practicals, 32 total hours)**: review of polynomials, exponential and logarithm functions; least squares method and linear regression; recognition of distributions and parameter estimation using log and semi-log plots; elements of infinitesimal and differential calculus for functions of one real variable;
- **Mathematical Tools 2 (L1 Physics/Chemistry/Engineering Sciences (PCSI), practicals, 27 total hours)** : functions of variables in \mathbb{R}^2 and \mathbb{R}^3 : domain, level sets; differentiation in several variables, partial derivatives, gradient; closed and exact differential forms, search for a potential; curve parametrization and line integrals.

5 Research activity

My research to date has developed along two main and essentially independent lines. The first line is the development of a **numerical simulator for red blood cell dynamics** accounting for the action of an electric field. This part of the work took place mainly during the PhD years. The second line is the design and analysis of a **hybrid numerical scheme of arbitrary order supporting polytopal meshes** for first-order PDE systems satisfying symmetry and positivity assumptions (**Friedrichs systems**). This second research line is the subject of the ongoing postdoctoral work. One can identify four modules that summarize these research activities, each detailed in one of the sections below. For each module, we describe the motivation, give contextual details where necessary, and present the obtained results. We also outline perspectives for future development. The sections I would present during a job interview would be section 5.2 and section 5.4.

5.1 Implementation of the Ghost Fluid Method in YALES2BIO

Motivation

The starting point is the biological tissue simulator **YALESBIO**, developed at IMAG and based on the **YALES2** kernel developed at CORIA. The goal of the project was to add to the simulator a module to represent the effect of an electric field on the elastic membrane of a red blood cell. We started from a consolidated electrodeformation model and searched for the most suitable numerical method to couple, for the first time, an **immersed boundary simulation** of red blood cells with the discretization of a discontinuous electric potential. This research is motivated, among others, by **applications in biomedical engineering** related to the design of automatic blood analyzers based on the Coulter principle (illustrated in figure 1).

Context

In **YALES2BIO**, the simulation of the flow around the cell is based on the Finite Volume method for solving the Navier–Stokes equations on a three-dimensional mesh of the considered geometry. The discretization of the red blood cell accounts for its spatial extent and the mechanical properties of its membrane. In particular, a cell is represented by introducing a triangulation of its surface. Communications between the flow volume and the cell membrane are implemented according to the principles of the Immersed Boundary Method [11]. The interaction model between the cell and the electric field adopted for the project is based on the Leaky Dielectric model [12], consolidated in the literature for systems similar to red blood cells. At the core of the model we introduce an additional variable representing the electric potential in the considered volume. The electric potential satisfies transmission conditions at the interface between the interior and the exterior of the cell, and its determination takes the form of an **elliptic interface problem**: in a domain Ω , split into two subdomains Ω_{int} , Ω_{ext} by an interface Γ with outward normal n_Γ , determine $u : \Omega_{\text{int}} \cup \Omega_{\text{ext}} \rightarrow \mathbb{R}$ satisfying

$$-\nabla \cdot (\sigma_\bullet \nabla u_\bullet) = f \quad \text{in } \Omega_\bullet, \bullet \in \{\text{int}, \text{ext}\}, \quad (1a)$$

$$[u]_\Gamma = J_\Gamma \quad \text{on } \Gamma, \quad (1b)$$

$$[\sigma \nabla u]_\Gamma \cdot n_\Gamma = \Phi_\Gamma \quad \text{on } \Gamma, \quad (1c)$$

$$u_{\text{ext}} = 0 \quad \text{on } \partial\Omega, \quad (1d)$$

where the brackets $[\cdot]_\Gamma$ denote the jump across Γ , $\sigma \in L^\infty(\Omega)$ is a conductivity coefficient constant on each subdomain and discontinuous across Γ , $f \in L^2(\Omega)$ and $J_\Gamma, \Phi_\Gamma \in L^2(\Gamma)$ express source terms.

Results

The main challenge of the project was to identify a numerical strategy for solving problem (1) compatible with the computing infrastructure of **YALES2BIO**, characterized by a non-conforming interface with respect to the computational mesh. The main outcome was the implementation of the **Ghost Fluid Method** [8], consolidated for multiphase flow simulation. At the end of the project, **YALES2BIO** includes a new module implementing the method, enabling representation of the discontinuity of the potential across the membrane (figure 2). The module is compatible with the parallelization procedures already present in **YALES2BIO**. The results of this project are detailed in the PhD thesis (<https://theses.hal.science/tel-05305250v1>)

Outlook

The follow-up of this project was the development of the polytopal approaches described in sections 5.2 and 5.3. These were so far implemented in 2D in a fundamentally sequential and non-optimized Python code.

Integrating these techniques into an industrial-scale platform like YALES2BIO would allow configuring 3D tests on configurations of applied interest, such as a blood analyzer. This is a project of strong interest, which could benefit from collaboration with a research engineer specialized in high-performance scientific computing.

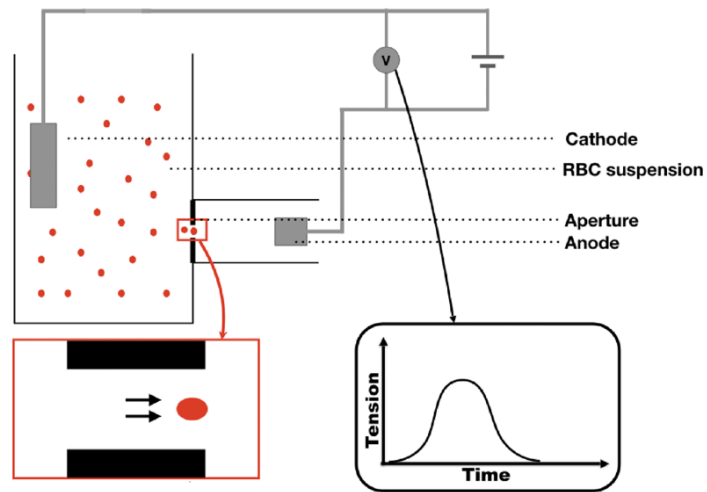
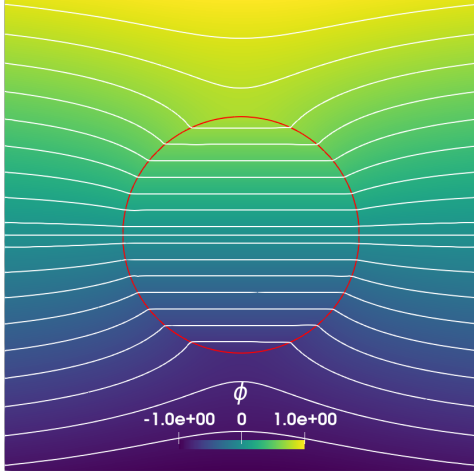
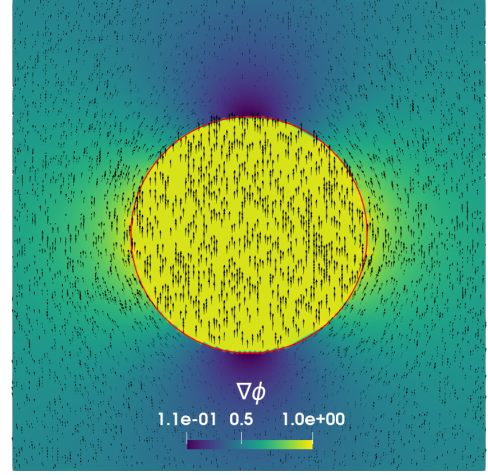


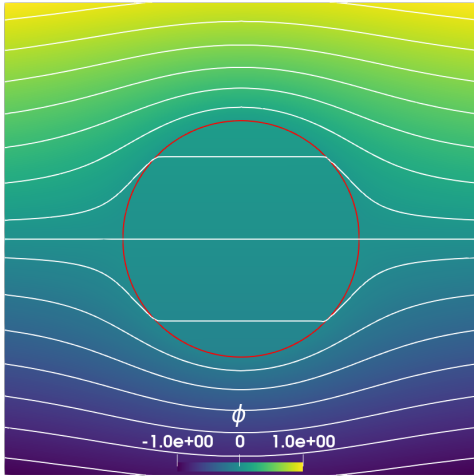
Figure 1: Section 5.1: schematic illustration of the operation of an analyzer based on the Coulter principle [2]. A blood sample is inserted into a solution contained in a system of communicating vessels (RBC suspension). A pressure difference drives the cells in the sample through a small measurement channel (Aperture) where a strong electric field is imposed by two electrodes (Anode/Cathode). The temporary perturbation of the measured voltage enables a quick cell counting procedure.



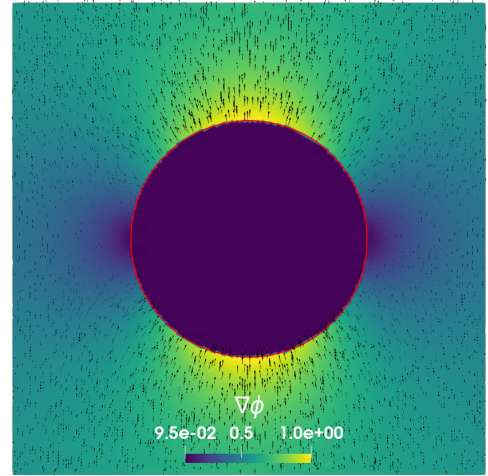
(a) $\phi_h, (\sigma_{\text{int}}/\sigma_{\text{ext}}) = 0.1$



(b) $\nabla_h \phi_h, (\sigma_{\text{int}}/\sigma_{\text{ext}}) = 0.1$



(c) $\phi_h, (\sigma_{\text{int}}/\sigma_{\text{ext}}) = 10$



(d) $\nabla_h \phi_h, (\sigma_{\text{int}}/\sigma_{\text{ext}}) = 10$

Figure 2: Section 5.1: numerical approximation of an electric potential solving (1) obtained in YALES2BIO, using the GFM. With $f = 0$, $\Phi_\Gamma = 0$, $J_\Gamma = 0$, the solution is characterized by the sign of $k = (\sigma_{\text{int}}/\sigma_{\text{ext}} - 1)$. On the first row, the numerical solution ϕ_h corresponding to $\sigma_{\text{int}}/\sigma_{\text{ext}} = 0.1$. On the second row, the solution for $\sigma_{\text{int}}/\sigma_{\text{ext}} = 10$.

5.2 A Discrete De Rham scheme for an elliptic interface problem

Motivation

We revisited problem (1) and developed a numerical scheme based on **interface-fitted meshes**. This project is motivated by the difficulty of correctly estimating the gradient jumps observed with the numerical strategies implemented in YALES2BIO. The proposed scheme is based on design principles presented in [5] and belongs to the class of **polytopal methods**. One of the main reasons to choose this framework is the flexibility offered by supporting generic polygonal meshes for **problems with an immersed boundary**. An interface-fitted mesh can be easily generated by cutting an initial mesh along the interface (figure 3). Since the interface resolution is independent from that of the background mesh, the scheme is well suited to approximate solutions where the boundary is curved without a significant increase of degrees of freedom in the volume.

Context

To define the scheme, we consider, for $\bullet \in \{\text{int}, \text{ext}\}$, a polygonal partition \mathcal{T}_h^\bullet of Ω^\bullet , as well as the sets \mathcal{E}_h^\bullet and \mathcal{V}_h^\bullet collecting respectively the edges and vertices of elements in \mathcal{T}_h . For a polynomial degree $k \geq 0$ arbitrary, we define the spaces

$$\underline{V}_{\bullet,h}^k := \left\{ \underline{v}_h = ((v_T)_{T \in \mathcal{T}_h^\bullet}, (v_E)_{E \in \mathcal{E}_h^\bullet}, (v_V)_{V \in \mathcal{V}_h^\bullet}) : \begin{aligned} v_T &\in \mathcal{P}^{k-1}(T) \text{ for all } T \in \mathcal{T}_h^\bullet, \\ v_E &\in \mathcal{P}^{k-1}(E) \text{ for all } E \in \mathcal{E}_h^\bullet, \\ v_V &\in \mathbb{R} \text{ for all } V \in \mathcal{V}_h^\bullet \end{aligned} \right\},$$

where $\mathcal{P}^k(X)$ is the space of polynomials of order up to k defined on X . The discrete space for seeking the solution is

$$\underline{V}_h^k := \underline{V}_{\text{int},h}^k \times \underline{V}_{\text{ext},h}^k,$$

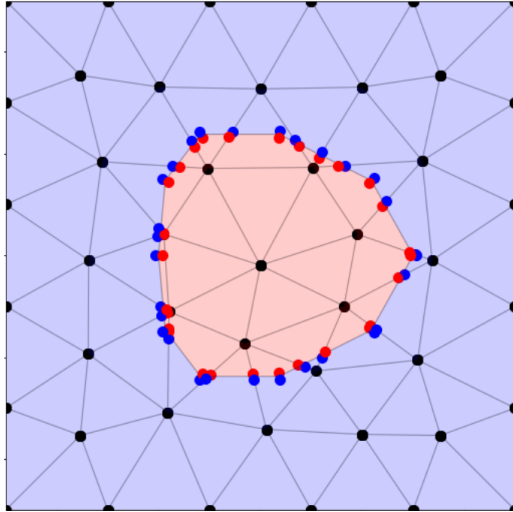
as well as the subspace $\underline{V}_{h,0}^k$ with zero values on $\partial\Omega$. The discrete space is characterized by hybrid degrees of freedom associated with geometric entities of different dimensions. The doubling of degrees of freedom at the interface allows representing discontinuous solutions. To define in a discrete sense the operators appearing in the weak form of (1), we introduce local reconstructions on each element.

Results

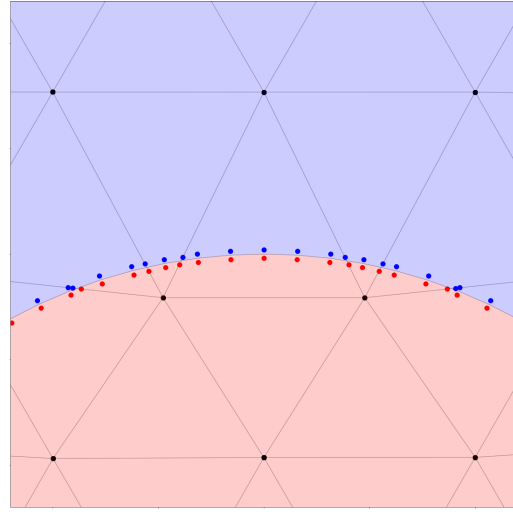
For this method, we provided a **stability analysis** and an **a priori error estimate** in terms of the mesh diameter and the chosen polynomial degree. In particular, we showed that the convergence error in a norm penalizing derivatives up to order 1 is controlled by a factor $h^{(k+1)}$, h being the diameter of \mathcal{T}_h , if the continuous solution is sufficiently regular. The stability of the scheme is independent of the conductivity ratio. The scheme was adapted to represent the temporal evolution of the transmembrane potential according to the model proposed in [13]. The detailed results of this module are the subject of [6] (<https://doi.org/10.1016/j.jcp.2025.113920>).

Outlook

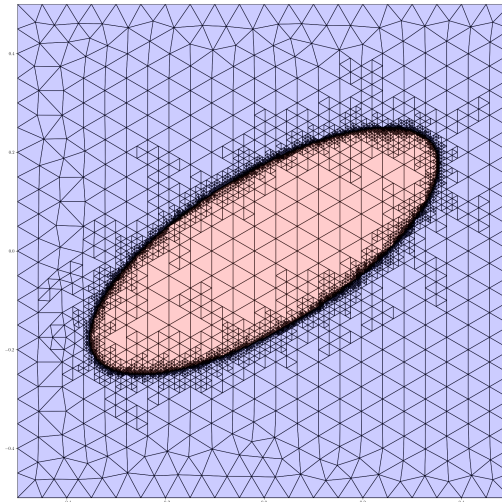
It would be interesting to study how the simple treatment of transmission conditions and the ability to accurately represent variable jumps could generalize. Transmission conditions across an immersed interface are a feature of other models and are not limited to electric potential descriptions. It would be interesting to evaluate how the method could adapt to modeling one-dimensional objects such as fractures in geologic contexts or cilia in a biomedical context. A follow-up to this project is described in section 5.3, and it is related to a moving interface model.



(a) Polygonal mesh obtained by cutting a simplicial mesh along an immersed boundary



(b) Detail of an interface with fine resolution relative to the background mesh



(c) Nonconforming refinement of elements close to the interface

Figure 3: Section 5.2; example of generic meshes supported by the Discrete De Rham method. In the generated mesh, data and the numerical solution can exhibit jumps only across element boundaries.

5.3 A capsule simulator supporting polygonal meshes

Motivation

The objective is to extend the polygonal discrete framework introduced in section 5.2 to the full simulation of red blood cells. To this end, we aim to introduce a simulator for the flow of a mobile elastic capsule immersed in a fluid environment based on a mesh that adapts to the interface at each time step.

Context

As a reference model, we consider the one presented in [1]. In this 2D description, the capsule is represented as a flexible membrane separating two fluid phases, and the flow is described by the Stokes equations. The velocity and pressure are determined as the solution of a **boundary value problem with transmission conditions at the interface**. We denote by Ω the domain, and we seek $v : \Omega \rightarrow \mathbb{R}^2$ and $p : \Omega_{\text{int}} \cup \Omega_{\text{ext}} \rightarrow \mathbb{R}$ such that

$$\nabla \cdot (\mu(\nabla v + \nabla^T v) - p\mathcal{I}_d) = 0 \quad \text{in } \Omega_{\bullet}, \bullet \in \{\text{int}, \text{ext}\}, \quad (2a)$$

$$\nabla \cdot v = 0 \quad \text{in } \Omega_{\bullet}, \bullet \in \{\text{int}, \text{ext}\}, \quad (2b)$$

$$[v] = 0 \quad \text{on } \Gamma, \quad (2c)$$

$$[\sigma] \cdot n_{\Gamma} = t_{\Gamma} \quad \text{on } \Gamma, \quad (2d)$$

$$\int_{\Omega} p = 0, \quad (2e)$$

$$v = g \quad \text{on } \partial\Omega, \quad (2f)$$

where $\mu \in L^{\infty}(\Omega)$ is a kinematic viscosity coefficient discontinuous across the interface, $t_{\Gamma} \in L^2(\Gamma, \mathbb{R}^2)$ represents a membrane tension distribution and $g \in H^{1/2}(\partial\Omega, \mathbb{R}^2)$ a boundary value. To numerically solve problem (2), we adapt a **Hybrid High-Order (HHO)** scheme presented in [3], modified to incorporate the source term t_{Γ} . The velocity unknown is sought in the hybrid space

$$\underline{U}_h^k := \left\{ \underline{v}_h = ((v_T)_{T \in \mathcal{T}_h}, (v_E)_{E \in \mathcal{E}_h}) : v_T \in \mathcal{P}^k(T, \mathbb{R}^2), v_E \in \mathcal{P}^k(E, \mathbb{R}^2) \right\},$$

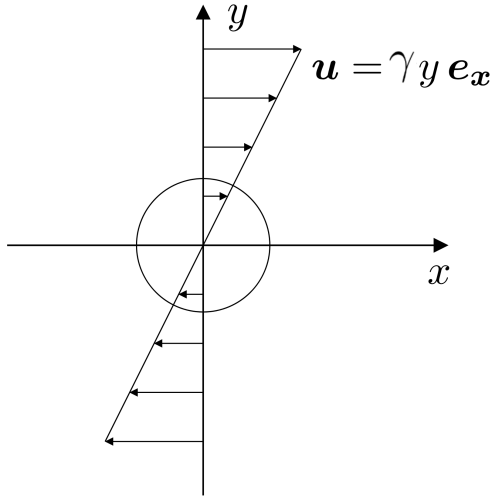
and the pressure is discretized in the space $P_h^k := \{p \in \mathcal{P}^k(\mathcal{T}_h) : \int_{\Omega} p = 0\}$. The choice is motivated by the ability of the scheme to support fitted meshes and by the possibility of statically condensing element variables to reduce the size of the linear system. The stability analysis and convergence properties of the adapted scheme remain unchanged. In particular, the error in an energy norm penalizing the velocity gradient converges like $h^{(k+1)}$. As for the hybrid scheme introduced for problem (1), the design of discrete operators is based on local reconstruction procedures. The Navier–Stokes solver is coupled with an explicit advancement scheme for the positions of the interface vertices.

Results

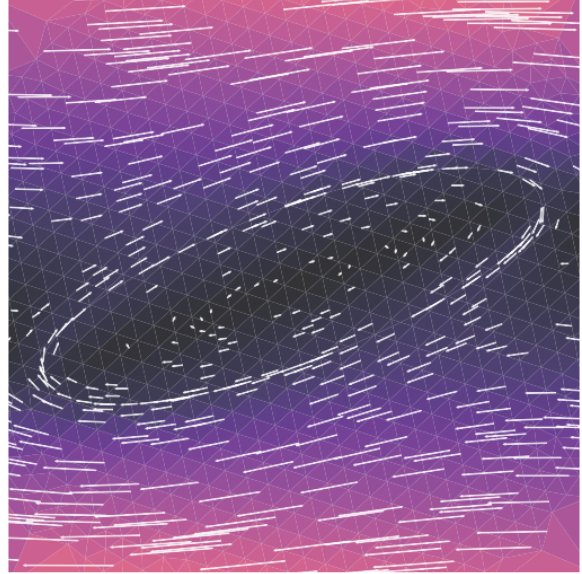
The scheme was tested on reference flow configurations (figure 4). The numerical simulation of capsules based on the HHO solver is the subject of a paper in preparation.

Outlook

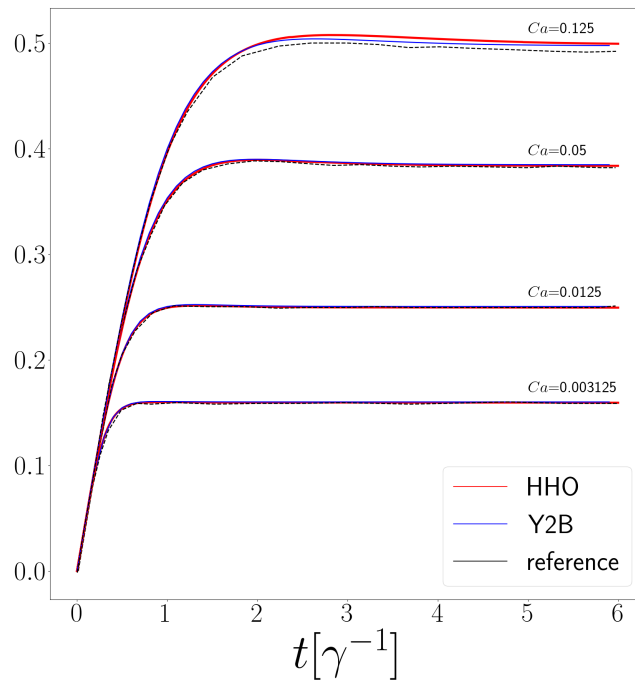
For this project, it would be interesting to perform a comparison with the most widespread approaches for numerically treating the surface/flow coupling. In particular, with the Immersed Boundary method implemented in YALES2BIO and the Boundary Integral method (see [1]), the two main references in the field. To this aim, it would be interesting to push the polytopal approach to its limits by showing on suitable configurations the advantages offered by the independent boundary resolution and the possibility to apply local refinement around the interface. This direction raises the question of an optimized implementation and the possible generalizations of the cutting algorithms to 3D. This project could benefit from the support of a research engineer.



(a) Illustrative scheme of the considered configuration (from [10]): circular capsule deformed by a velocity profile \mathbf{u} with shear rate γ .



(b) Maximum membrane deformation at equilibrium and velocity profile around the interface.



(c) Comparison of the capsule deformation ratio (major/minor axis) obtained with the three methods: black dashed: Boundary Integral method (reference), red: Immersed Boundary (YALES2BIO), blue: HHO solver. The comparison is repeated for different values of the capillary number Ca , representing the ratio between membrane elastic restoring forces and viscous forces.

Figure 4: Section 5.3: the reference configuration considered is a capsule initially at rest subjected to a linear shear. The result obtained with a simulator based on an HHO solver for problem (2) is compared with that obtained by a Boundary Integral method [1] and with the one obtained in YALES2BIO with the Immersed Boundary method.

5.4 Hybrid schemes for Friedrichs systems

Motivation

Friedrichs systems, introduced in [9], are a class of first-order linear PDE systems providing a common framework to treat **elliptic, hyperbolic or mixed-type equations**. Considering $m \in \mathbb{N}$ and a bounded polytopal domain $\Omega \subset \mathbb{R}^d$, let $\mathcal{K} \in L^\infty(\Omega, \mathbb{C}^{m \times m})$, and, for $i \in \{1, \dots, d\}$, $\mathcal{A}^i \in L^\infty(\Omega, \mathbb{C}^{m \times m}) \cap C^{0, \frac{1}{2}}(\Omega, \mathbb{C}^m)$, with $\mathcal{A}^H = \mathcal{A}$ almost everywhere in Ω , such that for some $\mathfrak{r}_b > 0$,

$$\mathcal{K} + \mathcal{K}^H - \sum_{i=1}^d \partial_i \mathcal{A}^i \geq 2\mathfrak{r}_b$$

almost everywhere in Ω . Let $\mathcal{N} \in L^\infty(\partial\Omega, \mathbb{C}^{m \times m})$ be defined as $\mathcal{N} = \sum_{i=1}^d \mathcal{A}^i n_i$, where n_i is the i -th Cartesian component of the outward normal to $\partial\Omega$. With $f \in L^2(\Omega, \mathbb{C}^m)$ and $\mathcal{M} \in L^\infty(\partial\Omega, \mathbb{C}^{m \times m})$, we seek $u : \Omega \rightarrow \mathbb{C}^m$ satisfying

$$\mathcal{K}u + \sum_{i=1}^d \mathcal{A}^i \partial_i u = f \quad \text{in } \Omega, \quad (3)$$

$$(\mathcal{M} - \mathcal{N})u = 0 \quad \text{on } \partial\Omega. \quad (4)$$

Among the examples of systems fitting the framework (3) are the **time-harmonic Maxwell equations** and the **diffusion-advection-reaction equation in mixed form** [7] with Dirichlet, Neumann or mixed boundary conditions. An existence and uniqueness proof for weak solutions was provided in [7]. In the same work, a Discontinuous Galerkin type scheme for a generic Friedrichs system is introduced. The aim of the project is the **hybridization** of this scheme, allowing a reduction of the total number of degrees of freedom through a **static condensation** procedure that eliminates element unknowns.

Results

We presented a new numerical scheme based on the hybrid unknown space

$$\underline{U}_h^k := \{ \underline{v}_h = ((v_T)_{T \in \mathcal{T}_h}, (v_F)_{F \in \mathcal{F}_h}) : v_Y \in \mathcal{P}^k(Y; \mathbb{C}^m) \text{ for all } Y \in \mathcal{T}_h \cup \mathcal{F}_h \},$$

where \mathcal{T}_h is a set of polytopes forming a mesh of Ω , \mathcal{F}_h is the set of all faces of the polytopes in \mathcal{T}_h and k is an arbitrary polynomial degree. The **inf-sup stability** of the method is ensured by a consistent stabilization term inspired by an upwind-like penalization introduced in [7]. The convergence of the numerical solution to the exact one with an **order** $h^{(k+\frac{1}{2})}$ is proved in a norm that includes the **L^2 norm of the directional derivative** and the constants appearing in the error estimate are made explicit with respect to the equation coefficients. The method was numerically tested for two examples of Friedrichs systems in 3D: a scalar diffusion-advection-reaction equation and a vectorial equation appearing in magnetohydrodynamics to describe diffusion/advection of the magnetic field. This work is reported in [4](<https://arxiv.org/abs/2602.10890>).

Outlook

One of the objectives of the ongoing postdoc is the specialization to the case of **two-block systems**, in which the variable is decomposed into a potential/flux pair. This applies in particular to second-order equations written in mixed form. Under standard assumptions on the structure of the matrix \mathcal{K} it is possible to write a scheme in which the flux variable is discretized in a space without face polynomials, allowing its static elimination. From the analysis point of view, it would be particularly interesting to identify superconvergence of the method under assumptions on the local regime, by introducing cellwise dimensionless numbers describing the relative magnitude of the equation terms. Other direct outcomes would be time discretization of a hyperbolic system as well as the integration of nonlinear terms.

6 Appendices

6.1 Pre-defense report by Ms Lisl Weynans



Rapport sur le mémoire de thèse présenté par Aurelio Spadotto pour l'obtention du grade de docteur de l'Université de Montpellier en Mathématiques et Modélisation

"Numerical simulation of red blood cell electrodeformation"

La thèse rédigée par Monsieur Aurelio Spadotto est un travail de recherche qui porte sur la simulation numérique de l'électrodéformation de globules rouges. Plus précisément, il s'agit d'étudier des méthodes numériques pour simuler les effets d'un champ électrique sur la déformation de la membrane d'une interface souple séparant deux phases fluides aux propriétés électriques contrastées. Une des applications visées par ce travail concerne le fonctionnement de dispositifs appelés cytomètres servant à caractériser des particules en suspension dans un fluide, comme par exemple des globules rouges.

Le mémoire comprend deux grandes parties, chacune dédiée au développement d'un certain type de méthode numérique pour résoudre le problème électro-hydrodynamique de la déformation d'une membrane : une approche par maillage non-adapté, et une approche par maillage adapté. Je détaille ci-après rapidement les contenus de ces parties, divisées en chapitres, mais avant cela, je tiens à souligner la complexité particulièrement élevée du problème étudié, tant en terme de modélisation qu'en terme de simulation numérique. En effet l'interaction entre la membrane élastique et le fluide résulte à la fois d'effets hydrodynamiques et électriques, avec la nécessité pour être prédictif d'obtenir des résultats très précis sur et au voisinage de la membrane.

Le premier chapitre est une introduction à la problématique physique, c'est-à-dire les suspensions de capsules déformables telles que les globules rouges, notamment dans le contexte de cytomètres. Est également présenté le logiciel YALES2BIO, développé depuis de nombreuses années, qui permet à l'heure actuelle de simuler de telles suspensions, mais sans prendre en compte l'effet d'un champ électrique sur la déformation des capsules (ou globules rouges). La problématique générale de la thèse est alors présentée.

Le deuxième chapitre présente une revue sur la modélisation des globules rouges, la description des différentes déformations d'une membrane élastique ainsi que les principes de modélisation de l'interaction fluide-structure dans le cas d'une cellule, c'est à dire d'une membrane séparant deux fluides. Le modèle électrohydrodynamique (Leaky Dielectric Model) retenu dans le cadre de cette thèse, décrivant les effets d'un champ électrique sur la déformation de la membrane, y est aussi présenté.

Le troisième chapitre présente les méthodes numériques choisies pour résoudre le problème d'électrodéformation dans le logiciel YALES2BIO, aussi bien pour la partie pré-existante à la thèse, c'est-à-dire l'interaction purement hydrodynamique fluides-membrane, que la partie électrodynamique, qui constitue une des principales contributions de cette thèse. Les simulations dans

YALES2BIO repose sur l'utilisation de deux types de structures géométriques : la membrane, décrite comme un maillage lagrangien, et le volume dans lequel est immergée cette membrane, constitué d'un maillage eulérien. Les équations de Navier-Stokes sont résolues avec une méthode de prédiction-corréction basée sur des volumes finis pour la discrétisation spatiale. L'interaction fluide-membrane est calculée avec une méthode de type IBM (Immersed Boundary Method). La résolution des effets électriques et de la force associée se fait dans un module complémentaire à YALES2BIO : le module ESS. Il s'agit de résoudre un problème elliptique présentant des discontinuités au travers de l'interface que constitue la membrane, et pour cela c'est la méthode Ghost-Fluid, dans sa version pour volumes finis, qui est choisie. On présente parfois les méthodes de types frontières immergées sur maillages non-adaptés comme des méthodes à la discrétisation relativement plus "simple" que leurs équivalents sur maillages adaptés, mais dans le cas de phénomènes complexes comme celui considéré ici, le développement d'une méthode capable de simuler l'ensemble des interactions fait nécessairement appel à des techniques sophistiquées, notamment pour la communication entre les structures géométriques, et constitue une belle réalisation scientifique.

Dans le quatrième chapitre est présentée une série de tests numériques en deux et trois dimensions ayant pour but la validation et l'étude du comportement numérique de la méthode présentée dans le chapitre précédent. Le premier test concerne la partie purement électrique et permet de comparer la solution numérique à une solution exacte. Des tests de convergence sont fournis en annexe mais à mon sens ils font véritablement partie de ce chapitre. Les tests suivants concernent la déformation de capsules soumises à un champ électrique, au repos puis dans un écoulement. Les configurations testées sont nombreuses et intéressantes. Dans la conclusion de ce chapitre, l'auteur relève que si la méthode numérique permet de réaliser des simulations des phénomènes considérés, des problèmes de robustesse numérique apparaissent en dimension trois. Cela motive le travail réalisé dans les deux chapitres suivants, où un autre type de méthode numérique sera investigué.

Le chapitre cinq présente donc les premiers pas du développement d'une alternative sur maillage adapté à la méthode précédente : un schéma de Rham discret, basé sur les principes des méthodes HHO (Hybrid High-Order) qui permet d'utiliser des maillages polytopaux. Ce schéma permet de résoudre un problème elliptique avec discontinuités aux travers d'une interface, et son application au modèle précédemment évoqué pour l'électrodéformation (Leaky Dielectric Model). Une des contributions du travail d'Aurelio Spadotto, dans ce chapitre, est d'avoir pris en compte les conditions de transmissions au travers de l'interface dans le formalisme des méthodes de Rham discrètes. Le chapitre comprend une description de la construction du schéma numérique, des tests numériques, une preuve de stabilité et une estimation de convergence. Comme je ne suis pas spécialiste de ce type de méthodes, deux points m'ont paru particulièrement intéressants au regard du paysage des méthodes de type frontières immergées classiques : le fait que l'estimation d'erreur ne dépende pas du rapport entre les conductivités dans chaque milieu, et le fait que ce type de méthode autorise à se servir d'un maillage non-adapté et à redécouper les mailles contenant l'interface sans avoir de contrainte particulière sur la forme des éléments qui en résulte.

Le dernier chapitre est une courte introduction à la simulation avec un schéma de Rham discret de l'interaction entre une capsule et un fluide décrit par les équations de Stokes, sans que des effets électriques ne soit pris en compte. Des résultats préliminaires sont présentés, notamment le calcul de l'évolution de la déformation d'une capsule dans un écoulement cisailé, avec une comparaison qualitative entre les résultats obtenus avec ce schéma de Rham discret et les résultats obtenus avec le logiciel YALES2BIO.

Ce manuscrit de thèse représente un travail très complet d'analyse numérique et de calcul scientifique appliqué à un problème complexe de couplage électro-dydrodynamique. Le fait que deux approches numériques très poussées mais complètement différentes aient été investiguées est particulièrement notable, et montre qu'Aurelio Spadotto maîtrise un grand nombre de techniques numériques. En lisant le manuscrit, j'aurais apprécié un peu plus de discussions et de commentaires sur les résultats numériques, par exemple les résultats des études de convergence, ainsi que sur la comparaison entre les deux méthodes, puisqu'il est rarement possible de pouvoir les comparer aussi directement que dans le cadre de cette thèse. Ce sont néanmoins des remarques minimes en regard de la qualité et de la quantité du travail accompli.

Pour ces raisons je donne un avis favorable et sans réserves à la soutenance de la thèse de M. Aurelio Spadotto.

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Talence, le 10/06/2025



6.2 Pre-defense report by Mr Alexei Lozinski



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Besançon, September 9th, 2019

Report on the PhD Thesis by Aurelio Spadotto

The manuscript by Aurelio Spadotto is a very interesting contribution to the field of numerical simulations of red blood cells in flow, in particular when they are subject to an electric field. The common theme uniting all the developments in the thesis, in mathematical terms, is the numerical approximation of boundary-value problems for the partial difference equations in the presence of internal interfaces, over which the solution (or its derivatives) can be discontinuous. These interfaces can evolve in time and their evolution can be a part of the problem. This theme comes up in two physical settings : 1) the fluid-structure interaction problem with the structure being the elastic membrane of a red blood cell, separating the fluid inside the cell from the ambient one; 2) the problem for the electric potential, when the electric field is applied to a red blood cell, leading to accumulation of the charge near its membrane, described via a jump of the potential near this surface. Two classes of numerical methods are applied to the two physical problems above : 1) low-order finite volume methods on a fixed grid, not fitted to the interface; 2) finite-element-type methods on polygonal grids matching the interface : the Discrete De Rham scheme and the Hybrid High-Order (HHO) method.

The thesis consists of two distinctive parts. The first one, i.e. Chapters 2-4, deals with the complete set of governing equations and the numerical methods of the first class mentioned above : finite volumes on a fixed grid, as implemented in YALES2BIO solver (a numerical software with massively parallel capabilities, implementing high order finite volumes schemes in complex geometries). The governing equations include those for fluid-structure interaction for the incompressible fluid modeled by Navier-Stokes equations and the thin structure modeled as an interface between two fluids (Section 2.3) ; a hyperelastic model for the structure (the membrane, Section 2.2) ; and the Leaky Dielectric Model for the electric forces (Section 2.4). This shows already the complexity of physical phenomena considered in the thesis. Given the diversity of these models, one cannot go into too much detail of each governing equation. The exposition is necessarily concise. I would appreciate however some more hints to the derivation of these equations, starting from the fundamental principles. As an example, the fluid-structure model could be presented through a variational (weak)

formulation involving well-defined volume and surface integrals. Instead of this, the Author opts for a presentation in the language of delta-functions à la Peskin whose precise mathematical or physical meaning is not always clear.

The implementation of the discretization schemes the above mentioned equations, performed using the YALES2BIO solver, is presented in Chapter 3. The Navier-Stokes equations are discretized by lowest-order finite volumes on a possibly unstructured grid, not fitted to the interface (the membrane). The membrane is represented by a triangulated surface and its displacement is approximated by piecewise linear polynomials on this surface mesh. The interaction between the membrane and the fluid on both sides of it is approximated by replacing the delta functions (supported at a point) by regularized delta functions (supported on the balls of radius $\sim h$ the mesh size). This allows one to take into account the discontinuity in the normal derivatives of the velocity across the interface. The most innovative contribution in this part of the thesis is the adaptation of this methodology to the Leaky Dielectric Model for the electric potential. The additional difficulty here is that the potential itself (rather than its normal derivative) is discontinuous across the interface. One cannot thus approximate it by the finite volumes on a unfitted grid in the same way as described above for the fluid velocity. The author bypasses this difficulty by introducing artificial extensions of the potential on both sides of the interface, and thus doubling the discrete unknowns near the interface. The quality of the resulting method is attested by numerical experiments reported in Chapter 4, on the benchmarks of increasing complexity, incrementally introducing multi-physics behavior. Some mesh-convergence studies are also reported in Appendix A. This part of the thesis is the result of a huge amount of work on conceiving, implementing, and testing very complicated numerical methods, important for real-life applications. One cannot expect a full theoretical analysis of all these methods. I would appreciate however more hints at the proofs of consistency with the governing equations.

The second part of the thesis, i.e. Chapters 5&6, is very different in spirit from the first part. The numerical methods developed here are fully analyzed and justified theoretically, while applying them to the governing equations which represent the most important bricks of the full multi-physics model considered in the first part. In particular, the electrostatic transmission problem across a surface with a prescribed jump in the electric potential (the Leaky Dielectric Model) is considered in Chapter 5. A novel numerical method, based on meshes with polygonal (polytopal) elements of general form, is introduced there. This allows one to naturally take into account the interface, by cutting the background fixed mesh with a polygonal line fitted to the interface. The idea of the method, referred to as the Discrete De Rham scheme, is to look for the approximation having the Degrees of Freedom (DOFs) in the form of polynomials (of possibly high degree) attributed to both the cells and the edges of the mesh, plus scalar DOFs attributed to the vertices. The approximate gradient of the solution (or the approximate solution itself) are recovered from these DOFs by mimicking well chosen integration by parts-like formulas on the mesh cells and edges. These approximations are then substituted in the weak formulation of the governing equations, leading, after the addition of an appropriate stabilization, to a consistent and stable scheme, whose optimal convergence can be proven at any order of the underlying polynomials. At the lowest order (which is the only case, which is actually implemented in the thesis), the DOFs of the method are just the values at the vertices, giving an economical and optimally convergent method on general computational grids. The method is also proven robust with respect to the physical coefficients (the permittivity of the medium for example) that can vary abruptly across the interface. All these nice properties are illustrated by numerical experiments.

Chapter 6 outlines a possible future direction for the overall modeling framework : the extension of the techniques developed in Chapter 5 to a full flow-structure interaction solver on polygonal meshes. The method chosen here is HHO, which is close to the Discrete De Rham scheme, but more

studied already in the existant literature.

The thesis is concluded by a short Chapter 7 that synthesizes the findings of both parts and outlines a clear research plan for future work.

In summary, Aurelio Spadotto's thesis is an impressive contribution to the numerical modeling of multiphysics phenomena involving evolving internal interfaces, with applications in the dynamics of red blood cells. The manuscript contains a lot of original and interesting results on innovative numerical methods and their implementation.

This manuscript by Aurelio Spadotto is certainty acceptable for a PhD defense.



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6.3 Defense report



DOCTORAT DE L'UNIVERSITÉ DE MONTPELLIER
RAPPORT DE SOUTENANCE



*Le rapport doit être rédigé en français.
En complément, une traduction en anglais peut figurer sur le même rapport.
Dans le cadre d'une cotutelle, le rapport peut être rédigé uniquement en anglais.*

Nom et prénom du doctorant : SPADOTTO Aurelio

Titre de la thèse : Simulation numérique de l'électro-déformation de cellules sanguines. Application au diagnostic médical

École Doctorale : I2S - Information, Structures, Systèmes

Date de la soutenance : 1 juillet 2025

Président du jury : VINCENT MOUREAU (COMPLÉTER)

Membres du jury :

Nom	Signature	Nom	Signature
Simon MENDEZ		List WEYNANS	PROCURATION
Alexei LOZINSKI		Daniele DI PIETRO	
Jérôme DRONIOU		Vincent MOUREAU	

Les membres du jury attestent avoir pris connaissance de l'intégralité du rapport.

Le(s) Directeur(s) de thèse atteste(nt) ne pas avoir pris part à la décision.

Si le rapport comporte plusieurs pages ou s'il est rédigé sur un document distinct, il devra être paraphé sur chaque page et signé par le Président du jury.

Aurelio Spadotto a fait une présentation très claire et didactique de son travail de thèse sur la simulation de l'électrodéformation des globules rouges qui a permis au jury d'avoir un éclairage complémentaire au manuscrit sur son travail de thèse. Il a su proposer une présentation synthétique de ses résultats principaux mettant en lumière des détails techniques bien choisis. Ce travail consistait au développement d'un modèle d'électrodéformation basé sur les méthodes de l'état de l'art dans la plateforme logicielle YALES2BIO. La validation extensive de ce modèle a montré des limites importantes des méthodes de frontières immergées, ce qui l'a mené au développement de schémas numériques originaux pour les maillages polygonaux. Ce travail est donc par essence multi-disciplinaire à la croisée de la physique, de la modélisation et des mathématiques appliquées avec des applications visées bien identifiées.

Il a répondu aux nombreuses questions qui portaient sur l'ensemble des aspects tant théoriques, numériques et de modélisation montrant ainsi son excellente maîtrise de toutes les notions abordées.

Pour toutes ces raisons, le jury décerne à Aurelio Spadotto le grade de docteur de l'Université de Montpellier en Mathématiques et Modélisation.

6.4 Degree diploma certificate



Université de Montpellier

ATTESTATION DE REUSSITE AU DIPLOME

Le Vice-Président de la Recherche atteste que

le Doctorat en Mathématiques et Modélisation
a été décerné à

Monsieur AURELIO EDOARDO SPADOTTO
né le 9 juillet 1997 à MILAN (ITALIE)

au titre de l'année universitaire 2024/2025

Titre des travaux : Simulation numérique de l' électro-déformation de cellules sanguines. Application au diagnostic médical

Date de soutenance : 1 juillet 2025

Jury :
M. VINCENT MOUREAU, Président du jury, Directeur de recherche
M. JÉRÔME DRONIOU, Membre du jury, Directeur de recherche
M. DANIELE DI PIETRO, Co-Directeur de thèse, Professeur des universités
M. ALEXEI LOZINSKI, Rapporteur, Professeur des universités
M. SIMON MENDEZ, Directeur de Thèse, Chargé de recherche
Mme LISL WEYNANS, Rapporteur, Professeur des universités

Ecole doctorale : Information, structures, systèmes

Fait à Montpellier, le 9 juillet 2025

Pour le Président de l'Université de Montpellier
et par délégation,


Jacques MERCIER
Le Vice-Président de la Recherche
Jacques Mercier



N° étudiant : 22120324

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